LEAST COST DESIGN OF WATER DISTRIBUTION NETWORK UNDER DEMAND UNCERTAINTY BY FUZZY - CROSS ENTROPY METHOD

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ABSTRACT

Water distribution system modeling problems are associated with large number of variables which are uncertain in nature. The uncertainties are due to (i) formula used (ii) coefficients in the formula and (iii) imprecise knowledge of the values of various parameters. Usually uncertainty exists in nodal demands and the uncertainty associated with nodal demands has to be considered for better design of water distribution networks. A two phase methodology for the least cost design of water distribution network based on fuzzy set theory and cross entropy method is proposed. The uncertain demands are considered as fuzzy sets, and the diameters for each pipe is selected from the commercially available diameters by cross entropy method. The model coded in MATLAB is linked to EPANET tool kit for hydraulic simulation. The proposed methodology was tested on Hanoi water distribution network, and the solutions obtained are compared with well known deterministic solutions from literature. The methodology is found to be effective in dealing with uncertainty in input parameters represented as fuzzy sets, and also the discrete diameters are very well handled by cross entropy method.

Key Words : Water distribution network, Fuzzy sets, Cross entropy, Optimization, Hydraulic Simulation

INTRODUCTION

The vast majority of mathematical models in engineering use deterministic approaches to describe various processes and systems. However, all real life problems incorporate uncertainty in one way or another. Water distribution system modeling problems are associated with a large number of variables which are uncertain in nature. The uncertainties are due to formula used, coefficients in the formula, and imprecise knowledge of the values of various parameters. Uncertainty modeling contributes to manage the water supply in a better way by quantifying the uncertainties and determining the sources of significant errors. In the process of water distribution network design, demands at the network's consumer nodes are the most uncertain quantity. Probabilistic based methods require statistically determined parameters for the representation of uncertainty in parameters which require large number of data. Besides statistically based uncertainty and sensitivity analyses, fuzzy set theory provides an alternative way that does not require crisp statistical measures of input parameter distributions. In fuzzy set approach, uncertainty is represented by fuzzy set parameters, which do not have direct correspondence with statistical background, though, indications of uncertainty can be easily recognized[1].

Babayan et al.[2] presented a methodology for the least cost design of water distribution networks under uncertain demand. The design problem is formulated as a stochastic, constrained single-objective optimization problem. The problem is solved using GAs after converting the original problem formulation to an equivalent, simplified deterministic optimization problem using standard deviation as a natural measure of the

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variability of nodal pressure heads caused by uncertainty in demands.

Rubinstein[3] proposed the Cross Entropy (CE) method, which was motivated by an adaptive algorithm for estimating probabilities of rare events in complex stochastic networks, which involves variance minimization. The CE method was applied successfully to solve the traveling salesman problem. Perelman and Ostfield[4] presented an adaptive stochastic algorithm for water distribution systems optimal design based on the heuristic cross-entropy method for combinatorial optimization.

AIMS AND OBJECTIVES

The study aims at developing a mathematical model to minimize the cost of the water distribution network under demand uncertainty using fuzzy - cross entropy method by selecting suitable diameters from the commercially available diameters, subject to satisfying several constraints such as nodal flow continuity equation, minimum nodal pressure requirements, loop head loss relations etc.

MATERIAL AND METHODS

The present study illustrates the application of fuzzy set theory and cross entropy method to WDS modeling problems, and the methodologies of fuzzy set theory and cross entropy are explained below.

Fuzzy Parameters and Membership Functions

Fuzzy set theory was first proposed by Zadeh in pioneering work in the field of system theory[5]. Since then it has become an important research tool that is used in a number of engineering fields, including decision making[6], and in the hydraulic engineering field, it has successfully been used in problems of optimization, risk analysis, and resource management.[7]

The central point of fuzzy formulation is the concept of a fuzzy set[8]. The fundamental principle of fuzzy set theory is the replacement of the concept that a parameter has a precise crisp value by the concept that a parameter can be fuzzy and can take on a range of values, with each of those values having a degree of membership. The membership function is not a probability distribution function but it shows possibility distribution. The possibility of a value increases as the value moves from zero to one.

Fuzzy sets in practice are often understood as fuzzy numbers and are represented through membership functions[9]. The most common types of membership functions for fuzzy numbers are : (i) Triangular; (ii) Trapezoidal. A fuzzy parameter x can be represented by a triangular or by a trapezoidal function as shown in **Fig. 1 (a)** and **Fig. 1 (b)** respectively. The value of the fuzzy parameter x is shown along the x-axis and its membership value, $\mu_A(x)$ is shown along the y-axis.

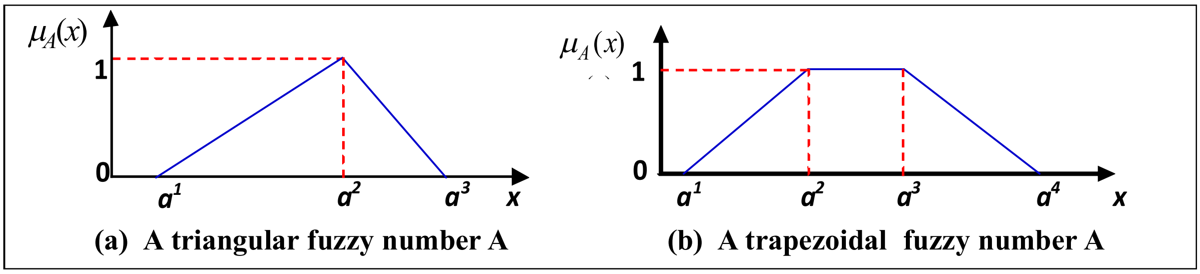

Fig.1: Fuzzy membership functions

Fuzzy ∝ cut technique

This technique uses fuzzy set theory to represent uncertainty or imprecision in the parameter(s). **Fig. 2** shows an uncertain parameter *P* represented as a triangular fuzzy number with support of A_0. The wider the support of the membership function, the higher the uncertainty. The fuzzy set that contains all elements with a membership of ∝ ε [0,1] and above is called the α -cut of the membership function. At a resolution level of α, it will have support of A_α. The higher the value of α, the higher the confidence in the parameter[1].

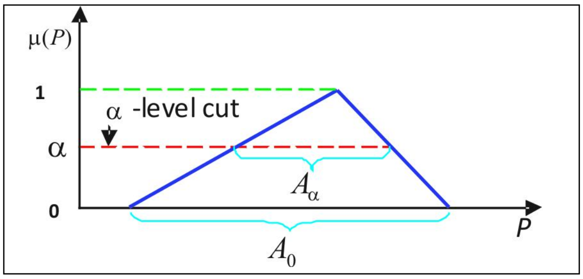

Fig. 2 : Fuzzy number, its support and α-cut

The method is based on the extension principle, which implies that functional relationships can be extended to involve fuzzy arguments and can be used to map the dependent variable as a fuzzy set. The membership function is cut horizontally at a finite number of α-levels between 0 and 1. For each α - level of the parameter, the model is run to determine the minimum and maximum possible values of the output. This information is then directly used to construct the corresponding fuzziness (membership function) of the output which is used as a measure of uncertainty.

Fuzzy Decisions and Fuzzy Programming

For a fuzzy problem, the fuzzy objective and constraints are characterized by their membership functions. Fuzzy goal *G* and fuzzy constraint *C* are fuzzy sets on the set of alternatives X, which

$$\mu_G : X \rightarrow [0,1]; \mu_c : X \rightarrow [0,1] \qquad (1)$$

are characterized by their membership functions[6]. The concept of decision may be stated as Confluence of goals and constraints i.e, fuzzy decision, D=G ∩ C and its membership func-

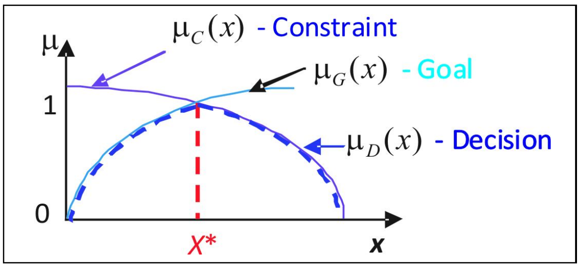

Fig.3: Membership function for the fuzzy decision making

tions are characterized in **Fig. 3**.

The Max-Min concept of decision making is used for the model formulation. Maximizing decision is to find the alternative X which maxi-

$$\mu_D(x^*) = \max[\min(\mu_G(x), \mu_C(x))], x \in X \qquad (2)$$

mizes μ_D(x).

Cross Entropy Method

The cross entropy method is an evolutionary iterative technique based on the concept of rare events, which involves two main stages : (i) Generation of a sample of random data according to a specified random mechanism and (ii) parameter updating of the random mechanism, on the basis of the generated data, so as to produce a "better" sample at the next iteration. The method derives its name from the cross entropy (or Kullback - Leibler) distance- a well known measure of information.

Entropy and cross entropy

Entropy can be termed as a measure of uncertainty associated with a process (measure of expected information gain from a random variable)[10]. The probability distribution of events if known provides a certain amount of information. Shannon defined a quantitative measure of the distribution in terms of entropy,

$$H(X) = -K\sum_{i=1}^{n} p_i \ln p_i \qquad (3)$$

called Shannon entropy given by the Equation 3. where *H(X)* represents the Shannon entropy corresponding to the random variable *X, K* is the Boltzmann constant and P_i represents the probability distribution corresponding to the variable x_i. The uncertainty can be quantified with entropy taking into account all different kinds of available information. Thus entropy is a measure of uncertainty represented by the probability distribution and is a measure of the lack of information about a system. If complete information is available, entropy is equal to zero, otherwise it is greater than zero.

Cross entropy is a distance measure from one probability distribution to another. One of the well known definitions of cross entropy is the Kullback - Leibler distance measure, serving to assess the similarity between two probability distributions: The statistical *p(x)* model and the true distributions *q(x)*. Cross entropy *[D(P,Q)]* is

$$D(P,Q) = \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} \qquad (4)$$

formulated as in Equation 4.

The interpretation of Equation 4 is that in order to estimate a probability distribution, the CE should be minimized. The goal is to find a distribution *p(x)* for which the Kullback - Leibler distance between *p(x)* and the true distribution *q(x)** is minimal, hence *p(x)** is the solution of the optimization problem.

Principle of Minimum Cross Entropy

According to Laplace's principle of insufficient reason, all outcomes of an experiment should be considered equally likely unless there is information to the contrary. Suppose a probability distribution for a random variable X is guessed as $Q = \{q_1, q_2, q_3, \ldots, q_n\}$ based on intuition or theory. This constitutes the prior information in terms of a prior distribution. To verify this guess, take a set of observations $X = \{x_1, x_2, x_3, \ldots, x_n\}$ and compute moments based on these observations. To derive the distribution $P = \{p_1, p_2, p_3, \ldots, p_n\}$ of X, all the given information and make the distribution as near to assumed intuition and experience as possible. Thus, the Principle Of Minimum Cross Entropy (POMCE) is expressed, when the cross entropy, D(P,Q) is

$$Minimize\ D(P,Q) = \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} \qquad (5)$$

minimized as in Equation 5.

On the basis of intuition, experience, or theory, a random variable may have an *apriori* probability distribution. Then, the Shannon entropy is maximum when the probability distribution of the random variable is that one which is close to the apriori distribution as possible. This is referred to as the principle of minimum cross entropy, which minimizes the Bayesian entropy[11]. This is equivalent to maximizing the Shannon entropy. Here minimizing *D (P,Q)* is equivalent to maximizing the Shannon entropy.

Cross Entropy Algorithm

The main steps involved in the cross entropy algorithm for solving WDN optimization problem is given below and the flowchart is as shown in **Fig. 4**.

1. Conversion of the WDN optimization problem to a stochastic node network.
2. Set the trial counter $t=0$ and assume equal probabilities for all the options as $P_{O,i}$.
3. Generate N_c sample vectors $X_v\ (x_1, x_2, \ldots, x_m)$ for $j = 1$ to N_c using the probability $p_{t,i}$ (i.e. generate a set of N_c possible vectors each of size *m*, and having zeros and ones, where one corresponds to choosing a specific network node, and zero otherwise). The value of N_c is taken as $N_c = \beta * nd$, where β is an integer value and *m* corresponds to the number of available options. The m dimensional vector $X_v\ (x_1, x_2, \ldots, x_m)$ has the discrete probability of $P = (p_1, p_2, \ldots, p_m)$.
4. Find out the performance function $PF(X_v)$ and check for constraints corresponding to each of the random vectors X_v, generated.
5. Now arrange the random vectors X_v, in the ascending order (if the problem is a minimization problem and descending order if it is a maximization problem) of their performance function $PF(X_v)$ values. Now the top most vectors will be having the best perform-

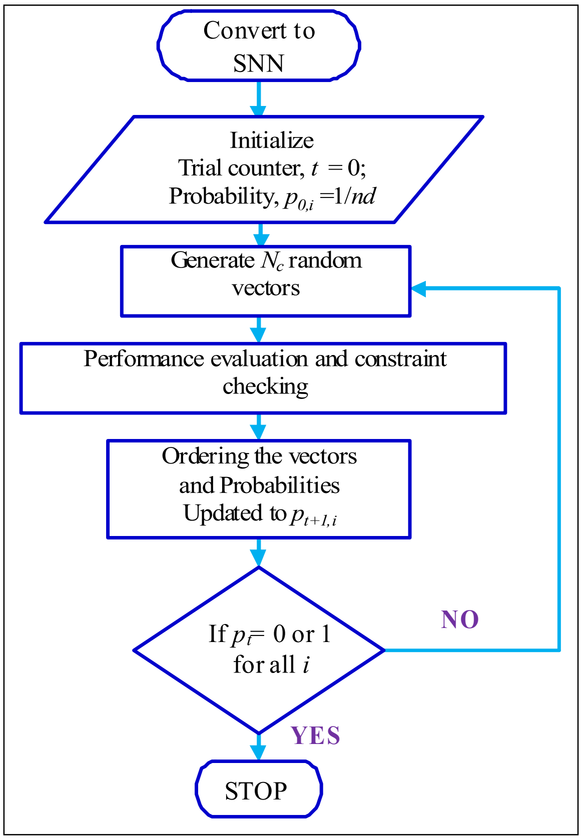

Fig. 4: Flow chart representing Cross entropy algorithm for WDN optimization

ance value and it is denoted as $PF(X_v) = \gamma^t$.

6. Choose a set (say ρ_c) of the top best performing vectors for updating the probability vector $p_{t,i}$ to the probability vector $P_{t+1,i}$. ρ_c corresponds to percentage of the vectors selected and its value varies between 10% and 20% but may change as a function of the sample size *N*. The i^{th} component of $p_{t+1,i}$ is obtained as given by the equation 6.

$$p_{t+1,i} = \frac{B_{t,i}}{TB_t} \tag{6}$$

where $p_{t+1,i}$, is the probability of success in the $(t+1)^{th}$ iteration of node *i*, $B_{t,i}$ is the total number of times node *i* was chosen (frequency) out of the best top performance vectors (i.e., TB_t the total number of vectors in the elite set) at iteration *t*.

In order to avoid early convergence (stopping criteria of decision variable probabilities approaching ZERO or ONE) to a local optimum solution, a smoothing parameter is used. The probability is modified as

$$p_{t+1,i} \leftarrow \alpha_c p_{t+1,i} + (1-\alpha_c) p_{t,i} \tag{7}$$

Using the above probability-updating scheme, the probability of choosing a node at each subsequent iteration increases as the frequency of occurrence of the node in the elite set increases. Updating the entire probability components using equation 6 in conjunction with the smoothing formula (equation 7) yields the new probability vector $p_{t+1,i}$. The main reason why such a smoothing updating procedure performs better is that it prevents the incidents of zeroes and ones in the reference vector, as in case such values are obtained they will remain permanently, which is obviously not required.

7. Check stopping conditions: If γ^t for subsequent iterations remains unchanged and if p_t converges to the degenerated case (i.e. all the probabilities p_t are close to zero and one) then stop. Declare the last γ^t as the optimal solution γ^* and its associated vector *X* as the design vector *X**, otherwise $p_t \leftarrow p_{t+1}$, and return to step 3.

RESULTS AND DISCUSSION

Model Formulation

The model is developed in two phases. In phase 1, a deterministic model is formulated to find the minimum and maximum values of the objective necessary for defining its membership function used in the fuzzy model formulated in phase 2. The detailed procedure for model development in phase 1 and phase 2 is given below.

Phase 1: Deterministic model

A deterministic model is formulated to minimize the cost of the pipe network, subject to satisfying the constraints such as minimum and maximum nodal demands, minimum nodal head requirements, and mass balance equation. This is to find the minimum and maximum values of the objective necessary for defining its membership function to in the fuzzy model which is use formulated in phase 2.

The mathematical model for WDN optimization

$$\text{Minimize } Cost = \sum_{i=1}^{n_p} C(d_i) * l_i \tag{8}$$

subject to,

$$H_j \geq H_j^{\min}, \quad \forall j \tag{9}$$

$$q_j^{in} - q_j^{out} - q_j = 0 \quad j = 1,2,3,.......,n_d \tag{10}$$

$$\left(\sum_{i=1}^{np_L} HL_i - \sum_{p=1}^{npu_L} h_p \right)_L = 0\,, L = 1,2,3,......., nL \tag{11}$$

where,

$$q_j^{in} = \sum_{i=1}^{n_{in}} Q_i \tag{12}$$

$$q_j^{out} = \sum_{i=1}^{n_{out}} Q_i \tag{13}$$

$$HL_i = \frac{\beta l_i Q_i^{1.852}}{C_{HW}^{1.852} d_i^{4.87}} \tag{14}$$

is as follows:

where $C(d_i)$ corresponds to the cost per length of the pipe having diameter d_i and l_i is the length of the i^{th} pipe, H_j and $H_j^{\min}$ are the available and minimum pressure heads at the j^{th} node; n_d = number of demand nodes; q_j^{in} = q_j^{out}

flow entering j^{th} node; = flow leaving from the j^{th} node ; q_j= demand at the j^{th} node; HL_i =head loss in i^{th} pipe; np_L= number of pipes in a loop; h_p = head raised by the pump p, npu_L= number of pumps in a loop; nL= number of loops in the WDN. n_{in} = number of incoming pipes to the j^{th} node; n_{out} = number of outgoing pipes from the j^{th} node; and Q_i = discharge or flow through the i^{th} pipe, β = constant depending on the units of head loss, length, diameter, and discharge; and C_{HW} = Hazen William's roughness coefficient.

The Cross Entropy (CE) method is used for the solution of the deterministic model in phase 1, In order to define the membership function for the objective (equation 8), phase 1 model has to be solved for two cases. In case 1, the model is to be solved with minimum values of the nodal demands to get the lower bound of the objective; and in case 2, the model is to be solved with maximum values of the nodal demands to get the upper bound of the objective. The bound values of the objective obtained in the deterministic model in phase1 can be used for defining the membership function of the objective in fuzzy model in phase 2.

The performance function used for solving the model is

$$PF(X_v) = \sum_{i=1}^{n_p} C(d_i) \times l_i + \sum_{j=1}^{n_n} PN \times MAX(0, H_j^{\min} - H_j) \quad (15)$$

where $PF(X_v)$ is the performance function for the solution vector used for the solution of the model, and PN is the penalty function rate for violating the nodal pressure constraint.

Phase 2: Fuzzy model formulation

The uncertainty in nodal demands is considered by describing them as fuzzy sets. Accordingly, the membership function for the constraint relating to the demands in each node is defined as shown in **Fig. 1 (b)**, using the bound values of the demands for each node. The membership function for nodal demands is described analytically by equation 16.

where q_i is the demand at the j^{th} node $q_j^1, q_j^2, q_j^3, q_j^4$ are trapezoidal fuzzy numbers for nodal demand at the j^{th} node.

The membership function for the objective func-

$$\mu(q_j) = \begin{cases} 0 & q_j < q_j^1 \\ \dfrac{q_j - q_j^1}{q_j^2 - q_j^1} & q_j^1 < q_j < q_j^2 \\ 1 & q_j^2 \le q_j \le q_j^3 \\ \dfrac{q_j^4 - q_j}{q_j^4 - q_j^3} & q_j^3 < q_j < q_j^4 \\ 0 & q_j > q_j^4 \end{cases} \quad (16)$$

tion Z is defined as μ (Z) as shown in **Fig. 5,** and analytically, the membership function for the objective μ (Z) is defined by equation 17.

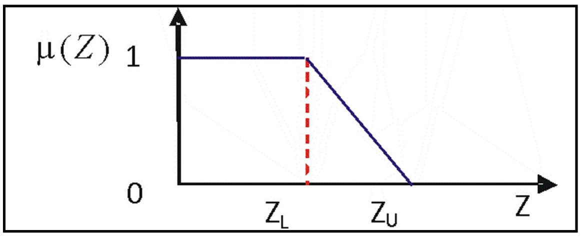

Fig. 5: Membership function for the objective

Analytically, the membership function can be described as follows,

where Z_L and Z_U are the minimum and

$$\mu(Z) = \begin{cases} 1 & \text{for } Z < Z_L \\ 1 - \dfrac{Z - Z_L}{Z_U - Z_L} & \text{for } Z_L \le Z \le Z_U \\ 0 & \text{for } Z > Z_U \end{cases} \quad (17)$$

maximum values of the objective function.

The fuzzy programming model for least cost is changed as follows :

The objective function changes to

$$\max \ \lambda \quad (18)$$

where λ is the level of satisfaction

The objective for cost as in Equation 8 changes as follows,

$$\lambda(Z_U - Z_L) + Z \le Z_U \quad (19)$$

The nodal mass balance equation (Equation 10) is transformed as

$$q_j^{in} - q_j^{out} - [\lambda(q_j^2 - q_j^1) + q_j^1] = 0 \ \text{ for } \ q_j^1 \le q_j \le q_j^2$$
$$j = 1,2,3,\ldots\ldots,n_d \quad (20)$$

$$q_j^{in} - q_j^{out} - [q_j^4 - \lambda(q_j^4 - q_j^3)] = 0 \ \text{ for } \ q_j^3 \le q_j \le q_j^4$$
$$j = 1,2,3,\ldots\ldots,n_d \quad (21)$$

and other constraints given by equations 9 and 11, and

$$0 \le \lambda \le 1 \qquad (22)$$

The final model is as follows :
The objective function is defined by Equation 18, and subject to constraints defined by Equations 9, 11, 19, 20, 21, and 22.

Model Application

The model developed is applied to Hanoi water distribution network[12], layout is shown in **Fig. 6,** is a real water distribution network constructed in Hanoi city at Vietnam, consists of 34 pipes and 32 nodes organized in three loops. The system is gravity fed by a single reservoir which is located at elevation of 100 m. The ground elevation for all nodes is 0. Data relevant to network like nodal elevation, pipe lengths, and trapezoidal fuzzy numbers for nodal demands are presented in **Table 1.** The system constraint is the minimum pressure head requirement for all nodes is restricted to 30 *m*. and no velocity constraint is taken into account. The available pipe diameters and unit cost of the pipes *(nd = 6)* for Hanoi WDN are given in **Table 2.** The solution search space of Hanoi WDN design problem is 6^{34}. The roughness coefficients of all the pipes are taken as 130.

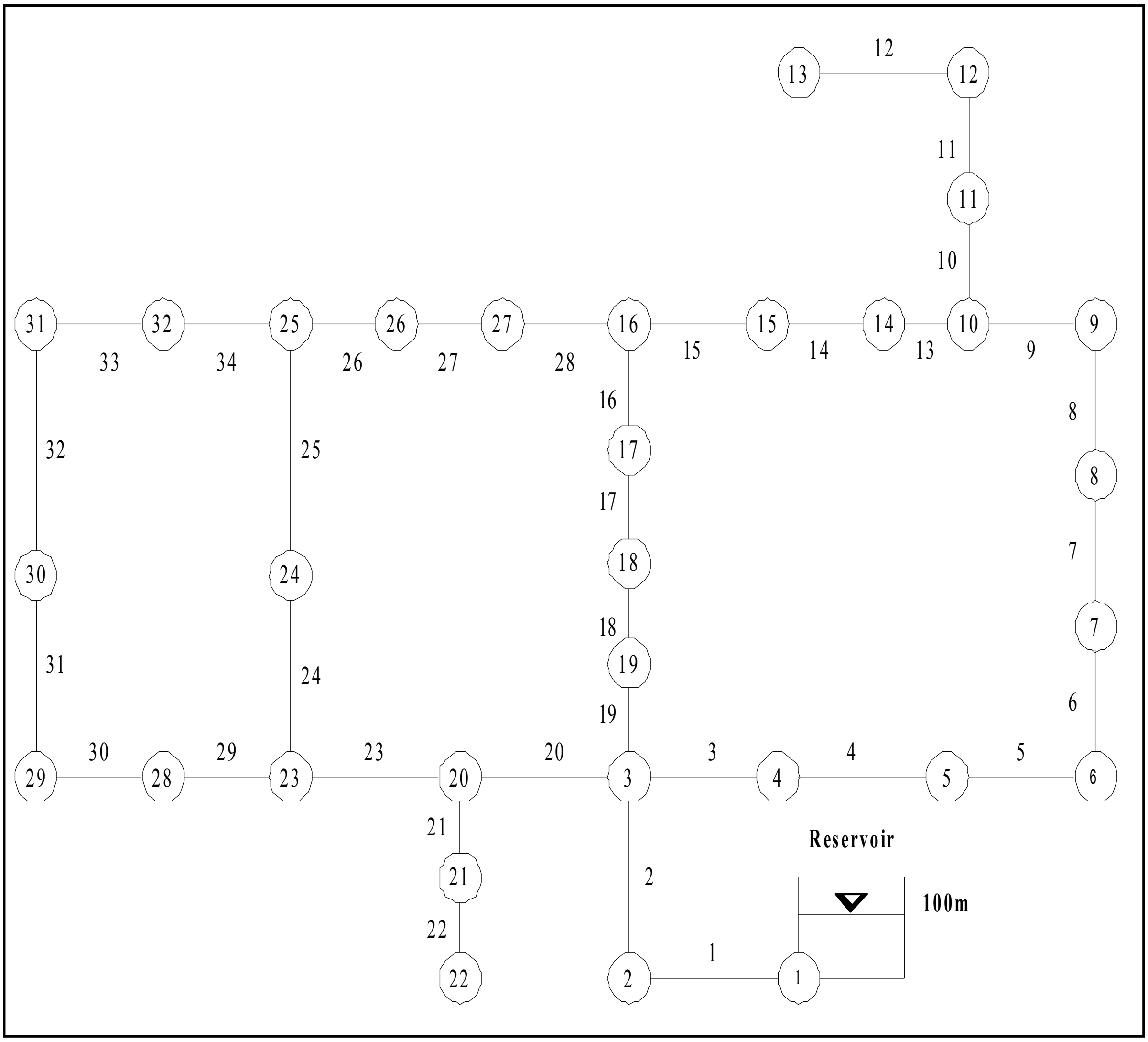

Fig. 6 : Layout of Hanoi WDN

Table 1: Input data of Hanoi WDN

Node No.	Elevation (m)	Fuzzy nodal demand (m^3/hr) q^1(min.)	q^2	q^3	q^4(max.)	Pipe No.	Length (m)
1	100*	-	-	-	-	1	100
2	0	797	885	890	985	2	1350
3	0	761	845	850	941	3	900
4	0	113	125	130	149	4	1150
5	0	648	720	725	803	5	1450
6	0	900	1000	1005	1111	6	450
7	0	1211	1345	1350	1491	7	850
8	0	491	545	550	611	8	850
9	0	468	520	525	583	9	800
10	0	468	520	525	583	10	950
11	0	446	495	500	556	11	1200
12	0	500	555	560	622	12	3500
13	0	842	935	940	1040	13	800
14	0	549	610	615	682	14	500
15	0	248	275	280	314	15	550
16	0	275	305	310	347	16	2730
17	0	774	860	865	957	17	1750
18	0	1206	1340	1345	1485	18	800
19	0	50	55	60	72	19	400
20	0	1143	1270	1275	1408	20	2200
21	0	833	925	930	1029	21	1500
22	0	432	480	485	539	22	500
23	0	936	1040	1045	1155	23	2650
24	0	734	815	820	908	24	1230
25	0	149	165	170	193	25	1300
26	0	806	895	900	996	26	850
27	0	329	365	370	413	27	300
28	0	257	285	290	325	28	750
29	0	320	355	360	402	29	1500
30	0	320	355	360	402	30	2000
31	0	90	100	105	121	31	1600
32	0	720	800	805	891	32	150
						33	860
						34	950

*Fixed nodal head at the source

Table 2: Commercially available diameters and unit cost for Hanoi WDN

S/N	Available diameter *inch*	*mm*	Unit cost (*$/m* length)
1	12	304.8	45.73
2	16	406.4	70.40
3	20	508	98.38
4	24	609.6	129.30
5	30	762.0	180.75
6	40	1016.0	278.28

RESULTS AND DISCUSSION

The lower and upper bounds of the cost obtained as 5.4475 million $ and 7.2617 million $ respectively, by solving the phase 1 model, which is used as bounds for defining the membership function for cost used in phase 2. On solving the phase 2 model, the optimum output of Hanoi WDN obtained is given in **Table 3** below, and also compared with the past study given in **Table 4.**

Table 3 : Optimum output of the Fuzzy-CE model for Hanoi WDN

Pipe No	Dia (inch)	Pipe No	Dia (*inch*)	Node No.	Available node pressure (*m*)	Node Demand (x10^3 m^3/h)	Node No.	Available node pressure (*m*)	Node Demand (x10^3 m^3/h)
1	40	18	30	1	100	-	18	58.9990	1.2877
2	40	19	40	2	97.3819	0.8505	19	64.2483	0.0529
3	40	20	40	3	64.9098	0.8120	20	57.7493	1.2205
4	40	21	20	4	61.3008	0.1201	21	49.1774	0.8889
5	40	22	20	5	56.8369	0.6919	22	48.7865	0.4613
6	40	23	30	6	52.2252	0.9610	23	44.8466	0.9994
7	30	24	24	7	51.1838	1.2925	24	38.6000	0.7832
8	40	25	24	8	46.5179	0.5237	25	35.8983	0.1586
9	30	26	16	9	45.6422	0.4997	26	36.1917	0.8601
10	24	27	24	10	43.1696	0.4997	27	36.6151	0.3508
11	24	28	24	11	38.9335	0.4757	28	40.0040	0.2739
12	24	29	16	12	35.7878	0.5334	29	32.6484	0.3412
13	12	30	12	13	31.9170	0.8985	30	33.0308	0.3412
14	20	31	12	14	37.1891	0.5862	31	33.1228	0.0961
15	16	32	20	15	37.2568	0.2643	32	33.9056	0.7688
16	24	33	20	16	38.4409	0.2931	Optimum level of satisfaction, $\lambda^* = 0.61$		
17	30	34	24	17	53.1408	0.8265			
					Optimum cost, Z* (x10^6\$)			6.1209	

The optimum cost is obtained as 6.1209 x 10^6 \$ corresponding to an optimum level of satisfaction of 0.61. The stopping criteria is arrived in 21,000 function evaluations with smoothing parameter α = 0.35, and PN = 1000000000. The solution obtained from the fuzzy - cross entropy model is satisfying the minimum pressure at all the nodes. On comparing the results of present study for Hanoi WDN problem with the results of Dijk et al.[14] for the same WDN with deterministic nodal demands, it is found that the optimum cost obtained is almost same and it can be seen that neglecting uncertainty in the design process may lead to underdesign of some of the pipes in the water distribution networks. Thus, the results obtained from the present study shows that the fuzzy- cross entropy method can be used for optimal design of WDN network, under uncertain demands.

Table 4 : Comparison of the results for Hanoi WDN

Pipe No.	Diameter(*inch*) Dijk et al.[14]	Diameter(*inch*) Present Study	Pipe No.	Diameter(*inch*) Dijk et al.[14]	Diameter(*inch*) Present Study
1	40	40	18	24	30
2	40	40	19	24	40
3	40	40	20	40	40
4	40	40	21	20	20
5	40	40	22	12	20
6	40	40	23	40	30
7	40	30	24	30	24
8	40	40	25	30	24
9	40	30	26	20	16
10	30	24	27	12	24
11	24	24	28	12	24
12	24	24	29	16	16
13	24	12	30	12	12
14	12	20	31	12	12
15	12	16	32	20	20
16	12	24	33	16	20
17	16	30	34	24	24
			Total cost (*\$*)	6.11×10^6	6.12×10^6

CONCLUSION

A mathematical model is developed based on fuzzy - cross entropy optimization for the least cost design of WDN, by considering the uncertainty in the nodal demands. subject to satisfying the hydraulic constraints. The model is developed in two phases. In phase 1, a deterministic model is formulated to find the lower and upper bound values of the objective by considering the minimum and maximum values of the uncertain nodal demands necessary for defining its membership function used in the fuzzy model formulated in phase 2.

The developed model is applied to Hanoi WDN, and the study shows that the fuzzy set theory is very much effective in handling uncertainty in input parameters, and the WDN problem can be successfully optimized by Cross entropy method with a few number of objective function evaluations, and efficient in handling discrete diameters.

REFERENCES

1. Nemanja B. and Marko I., Fuzzy approach in the uncertainty analysis of the water distribution network of BECEJ, *Civil Engrg. Environ. Systems,* **23**(3), 221-236, **(2006)**
2. Babayan A., Kapelan Z., Savic D. and Walters G., Least-cost design of water distribution networks under demand uncertainty, *J. Wat. Resour. Plang. and Mgmt.,* **131** (5), 375 - 382, **(2005)**
3. Rubinstein R.Y., Optimization of Computer simulation Models with Rare Events, *Europ. J. Operat. Res.,* **99**, 89-112, **(1997)**
4. Perelman L. and Ostfeld A., An adaptive heuristic cross-entropy algorithm for optimal design of water distribution systems, *Engi. Opti.,* **39** (4), 413 -428, **(2007)**
5. Zadeh L.A., Fuzzy sets, *Information and Control,* **8** (3), 338-353, **(1965)**
6. Zimmermann H.J., Fuzzy set theory and its applications, *Kluwer Academic Publishers Ltd., Dodrecht, Netherlands,* **(1991)**
7. Saad M., Bigras P., Turgeon A. and Duquette R., Fuzzy learning decomposition for the scheduling of hydroelectric power system, *Wat. Res.,* **32** (1), 179-186, **(1996)**
8. Dubois D. and Prade H., Fuzzy sets and system- Theory and applications. Academic, *San Diego, Calif.,* **(1980)**
9. Kwang H.L., First course on fuzzy theory and applications, Springer (India) Pvt. Ltd., New Delhi, India, **(2009)**
10. Shannon C.E., A mathematical theory of communication, *Bell Sys. Tech. J.*, **27** (3), 379 - 423, July, **(1948)**
11. Kullback S. and Leibler R.A., On information and sufficiency, *Ann. Math. Statics,* **22** (1), 79-86 **(1951)**
12. Fujiwara O. and Khang D.B., A two-phase decomposition method for optimal design of looped water distribution networks, *Wat. Res.,* **26** (4), 539-549, **(1990)**
13. Abebe A., Guinot V. and Solomatine D., Fuzzy alpha-cut vs. monte carlo techniques in assessing uncertainty in model parameters, *Proc, 4th Int. Conf. on Hydroinformatics,* Iowa City, USA, **(2000).**
14. Dijk M.V., Vuuren S.V. and Van Z., Optimizing water distribution systems using a weighted penalty in a genetic algorithm, *ISSN, Water SA*, **34** (5), 0378-0478, **(2008)**